NOTE ON BETA ESTIMATION FOR EMERGING MARKET SECURITIES

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Computing the security beta (sensitivity of security *i* to benchmark return) is not rocket science. All you need is a reasonable length of security returns (R_{it}), market returns (R_{mt}), and risk-free rate (R_{ft}). The beta estimated from OLS regression below is then used for computing the required rate of return.

$$R_{it} - R_{ft} = \hat{a}_i + \hat{\beta}_i \cdot (R_{mt} - R_{ft}) + \hat{\varepsilon}_{it}$$

The objective of this article is to address issues on beta estimation, while some of the problems, for example estimation error and nonsynchronous trading are not at all unique to emerging markets, I discuss a couple of other issues that we should bear in mind when computing and interpreting the value of the security beta. The following sections provide details on the issues and how to deal with them.

Estimation Error

Beta estimation, like any work involving social science data, is subject to sampling error. Noise from a single security beta is typically larger than that from characteristic-based portfolios. As the idiosyncratic components of individual stocks within a portfolio tends to cancel each other out, portfolio betas are estimated with higher precision. Unfortunately, forming portfolio betas may not be an option in all emerging markets. Being relatively small and full of young firms, some of these markets may have too few stocks in each portfolio to sufficiently reduce the idiosyncratic terms.

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The other caveat in using portfolio betas is that the user is throwing away a great deal of information found in the individual betas. This is particularly important when the user is examining cross-sectional differences.

An alternative approach is to use a Bayesian adjustment suggested by Vasicek (1973). The idea is to compute the individual security beta β_i and then the average beta $\overline{\beta}$ of all stocks and obtain both the standard error of the security beta σ_i and the cross-sectional standard deviation of all the estimated betas, $\sigma_{\overline{\beta}}$.

As shown in the equation in the box below, the adjustment gives a weight of 1.0 to the individual beta if the cross-sectional standard deviation is large. This occurs when the spread of each individual beta is sufficiently large meaning that that the values of individual betas are equally likely. In contrast, if the standard error of the security beta is extremely large relative to the cross-sectional standard deviation, then the weighted beta will be "shrunk" towards the grand mean.

Computing portfolio beta:

1. Form portfolio returns based on characteristics. The common approach is to form them based on size deciles. For markets with fewer stocks, you may want to use quintiles.
2. Compute the portfolio returns for each size decile then use the portfolio returns to obtain beta for each portfolio.

$$R_{pt} - R_{ft} = \hat{a}_i + \hat{\beta}_p \cdot (R_{mt} - R_{ft}) + \hat{\varepsilon}_{pt}$$

3. Assign the portfolio betas β_p to individual stocks in that portfolio.

Computing Vasicek (Adjusted) Beta

1. Compute individual security beta and average of all betas.
2. Compute adjusted beta, β^A from

$$\beta_i^A = w \cdot \beta_i + (1 - w) \cdot \overline{\beta}$$

where,

$$w = \frac{\sigma_{\overline{\beta}}^2}{\sigma_i^2 + \sigma_{\overline{\beta}}^2}$$

Nonsynchronous Trading

The problem of nonsynchronous or thin trading occurs when there is a mismatch between the market price and the recorded price of the stock since its last trade. This results in an upward bias in the betas of frequently traded shares and downward bias in infrequently traded shares. The intuition is the observed covariance between stock return and market return is related to stock trading frequency.

Dimson (1979) suggests a method of beta correction to accommodate thin trading. For shares that are infrequently traded, the leading beta coefficient, β^{+n} will be small compared to the lagged coefficient, β^{-n}. Including lagged coefficients becomes more important when infrequently traded shares are being regressed on value weighted index that is dominated by few large stocks. The method of coefficient aggregation raises the beta of the infrequently traded stocks while lowering those with frequent trade that dominates the index.

Computing Dimson beta

Estimate the following market model which includes observed leading, contemporaneous, and lagged returns, $R_{it} - R_f = \hat{a} + \Sigma_{-n}^{n} \hat{\beta}_k \left(R_{m\,t+k} - R_f\right) + \hat{\varepsilon}_t$, Dimson (1979) show that the true market risk of a security, $\hat{\beta}_i$ can be obtained from $\Sigma_{k=-n}^{n} \hat{\beta}_k$.

Dominant Security in Value-Weighted Index and Changing Correlation Structure

When a security or a group of securities consisting of few stocks dominate the market, there can be a downward bias of the betas of smaller securities. The reason is stocks with significant weights in the index will of course, covary more with the market. This creates problems with estimation and interpretation of the security beta.

The dominance of a few securities in an index makes inferences difficult. In theory, the benchmark market index should be a diversified group of assets. This is not the case if we try to compute the security beta of let's say stocks on the Sao Paulo stock exchange. Brazil's telecom stock, Telebras accounts for over 50% of the weight in the Sao Paulo Stock Index. Consequently, when we compute the sensitivity of the other securities, we are actually computing the sensitivity of the stock relative to Telebras as opposed to a diversified group of assets in our "wealth" portfolio. There is of course, nothing wrong with this if we assume that the representative investor holds more than 50% of his wealth in telecom.[1] Change in correlation structure between the security and the market also results in beta values that are unstable. This is particularly common when the compositions of emerging markets indices are constantly adjusted to accommodate new companies and elimination of dead stocks.

Table 1 presents a hypothetical case when the market is comprised of security *i* and the rest of the market, *m-i*. Security *i* has higher risk ($\sigma_i = 6\%$) than the rest of the market ($\sigma_{m-i} = 3\%$).

Table 1: Beta of Stock *i* and Weight in Market

Correlation	*Beta i*	*Beta m-i*	*Weight i*	*Weight m-i*	*Std i*	*Std m-i*
1.00	1.98	0.99	0.01	0.99	0.06	0.03
1.00	1.80	0.90	0.11	0.89	0.06	0.03
1.00	1.65	0.83	0.21	0.79	0.06	0.03
0.75	1.51	0.99	0.01	0.99	0.06	0.03
0.75	1.56	0.93	0.11	0.89	0.06	0.03
0.75	1.56	0.85	0.21	0.79	0.06	0.03
0.25	0.54	1.00	0.01	0.99	0.06	0.03
0.25	0.94	1.01	0.11	0.89	0.06	0.03
0.25	1.28	0.93	0.21	0.79	0.06	0.03
0.15	0.34	1.01	0.01	0.99	0.06	0.03
0.15	0.79	1.03	0.11	0.89	0.06	0.03
0.15	1.20	0.95	0.21	0.79	0.06	0.03

$$\beta_i = \frac{\operatorname{cov}(R_i, R_m)}{\sigma_m^2} \qquad (1)$$

$$\begin{aligned}\operatorname{cov}(R_i R_m) &= \operatorname{cov}(R_i, w_i R_i + w_{m-i} R_{m-i}) \qquad (2)\\ &= w_i \sigma_i^2 + w_{m-i} \operatorname{cov}(R_i, R_{m-i})\end{aligned}$$

[1] Somehow, this may not all together be a bad idea for the representative Thai investor given current leadership.

$$\beta_i = \frac{w_i \sigma_i^2 + (1 - w_i)\sigma_i \sigma_{m-i} \rho_{i,m-i}}{w_i^2 \sigma_i^2 + (1 - w_i)\sigma_{m-i}^2 + 2w_i(1 - w_i)\sigma_i \sigma_{m-i} \rho_{i,m-i}} \qquad (3)$$

Apparently, there is a great deal of variation in the value of β_i as we vary the weights of security *i*, w_i and the remaining *m-i* stocks in the market. Figure 1 plots the result in Table 1 and shows further variation in the weights.[2] Furthermore, the correlation between security *i* and *m-i* also determines β_i. In the case when there is high covariance between *i* and *m-i* (ie. $\rho_{i,m-i}$ = 1.0, 0.75), β_i then > 1 and $\beta_i > \beta_{m-i}$ for all $w_i \in (0,1)$. There is nothing striking about this result as higher risk means higher beta in a mean-variance sense. In contrast, when the covariance is low (ie. $\rho_{i,m-i}$ = 0.25, 0.15), $\beta_i < 1$ and $\beta_i < \beta_{m-i}$ then for $w_i < w_c$, given w_c is some positive weight. There is nothing unusual about this either. A smaller beta for a security with relatively larger variance than the rest of the market is permissible as it is how the additional security will help reduce overall market risk that is important rather than its own variance *per se*. Yet here lies the bump in the road. As noted earlier, what goes into the "rest of the market" is not a trivial matter. Consider instead that *i* represent a group of securities that are diversified, but account for small weight in the market while the rest of the market is dominated by a few stocks in similar industries that have high correlation with each other but low correlation with those in group *i*. In this situation, we will usually find that stocks in group *i* will have relatively lower beta than those few stocks that dominate the market. I illustrate this point using real market returns.

Table 2 presents the average beta of stocks classified by size deciles in three different markets, Korea, Malaysia, Taiwan, and Thailand.[3] The second column in Table 2 indicates the decile rank from small to large. In three out of four countries, beta is increasing in size. In contrast, standard deviation is decreasing in size as smaller stocks clearly have much higher return volatility than those with larger size. Malaysia's case is different as the average market cap in each size decile is more evenly dispersed. In other markets, stocks in the decile 10 (top larges stocks) dwarfs most stocks in other deciles.

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[2] The non-linearity from the curves comes from equation (3).
[3] Market and stock returns are from Datastream.

Table 2: Average Stock Beta by Size Decile

Beta computation is based on daily data between 1995-2003. STD is standard deviation of daily returns computed from squared of daily returns.

Decile	Korea beta	Malay Beta	Taiwan Beta	Thai Beta	Korea Std	Malay Std	Taiwan Std	Thai Std
1	0.87	1.17	0.46	0.69	6.49%	4.36%	2.06%	5.22%
2	0.98	1.06	0.64	0.68	5.85%	3.49%	1.92%	4.99%
3	0.98	1.09	0.51	0.65	5.11%	3.68%	2.35%	4.45%
4	1.16	1.07	0.68	0.94	5.25%	3.55%	1.88%	4.90%
5	1.09	1.23	0.6	0.91	4.70%	3.41%	1.98%	3.98%
6	1.14	0.90	0.69	1.21	4.35%	3.09%	1.73%	4.01%
7	1.13	0.94	0.89	0.98	3.94%	2.36%	1.90%	3.41%
8	1.12	0.98	0.7	1.24	3.94%	2.38%	1.64%	3.45%
9	1.41	1.14	1.00	1.25	3.52%	2.42%	1.74%	3.26%
10	1.59	1.01	1.11	1.63	3.60%	1.93%	1.59%	2.72%

An alternative approach is to compute betas based on an equal-weighted index or separate stocks into 2 groups, investable and non-investable. Many emerging markets are segmented into stocks that have high visibility and liquidity and those that do not. By computing both investable and non-investable betas we have a measure of the security return sensitivity to investable stocks returns, which typically dominates the market while the latter measures smaller stocks sensitivity relative to non-investable returns.

Computing investable and non-investable betas

1. Separate stocks into investable and non-investable groups. Lists of investable and non-investable stocks are readily available from the IFC emerging markets database. The user may also split non-investable stocks from the investable group by separating those in the lowest size deciles from the rest of the market.
2. Compute investable and non-investable portfolio returns.
3. Obtain investable and non-investable betas from equation,

$$R_{it} - R_{ft} = a_i + \beta_i^I \left(R_{pt}^I - R_{ft}\right) + \beta_i^N \left(R_{pt}^N - R_{ft}\right) + \varepsilon_{it}$$

Table 3 reports investable and non-investable betas. As expected, smaller stocks have higher sensitivity to non-investable returns, reflecting relative riskiness in their own class. The interesting part is the negative investable betas found in smaller stocks in all markets shown. The negative beta implies that an increase in investable stocks tends to reduce the returns of non-investable stocks. This is plausible if a rise in the overall market dominated by large stocks leads investors to flock into the investable group while selling out the non-investable stocks. (see Bae, Chan, and Ng (2002) for issues of investability and volatility).

Table 3: Investable and Non-investable Betas

	Korea		Taiwan		Thailand	
Decile	**IBETA**	**NBETA**	**IBETA**	**NBETA**	**IBETA**	**NBETA**
1	-0.45	1.43	-0.15	1.08	-0.08	0.91
2	-0.262	1.27	-0.13	1.12	-0.13	0.98
3	-0.19	1.17	-0.25	1.22	-0.22	1.2
4	-0.12	1.17	-0.03	1.06	0.09	0.94
5	-0.04	1.04	-0.03	1.02	-0.12	1.16
6	0.06	0.95	0.13	0.84	0.02	1.10
7	0.22	0.78	0.55	0.57	0.10	0.91
8	0.3	0.64	0.36	0.65	0.34	0.88
9	0.45	0.57	0.83	0.17	0.76	0.42
10	0.78	0.30	1.01	0.05	1.13	0.06

Short History, Structural Breaks, and Market Integration

Many stocks on the stock exchanges of developing economies have fairly short historical period. To complicate matters, these stocks undergo constant internal (company-specific) and external (market-wide) changes.

On one hand, we would like to utilize a long stretch of time series to compute the beta in order to improve estimation precision. On the other, the structural breaks that are frequently observed in emerging markets must also be taken into account. We allow for time-variation in the estimated beta by computing rolling betas instead of a single beta over the entire time period. The commonly used rolling period is 60 months. Thus for security *i*, time period 1 to 60, 2 to 61, ….,T-59 to T you obtain $\beta_1, \beta_2, ..., \beta_{T-59}$. A much longer rolling period is preferred for higher frequency data which tends to provide beta estimates that are less stable.

Another issue involving estimation of emerging market stock betas is the partial integration of these markets to the rest of the world (see Errunza and Losq (1983) and

Carrieri, Errunza, and Hogan (2002)). To cope with this matter, we can add world market return in our regression to obtain security *i*'s sensitivity to both local market return and world market return. Larger stocks tend to have larger and more positive exposure to world returns.

Conclusion

Precision in beta estimation improves the accuracy of the cost of capital. This article provides various approaches for refining the beta estimates. The adjustments help deal with issues of estimation error, thin trading, choice of benchmark index, and structural breaks and market integration. There is no hard and fast rule as to which method is best. The choice of the correction method depends on the context of the analysis and the characteristics of the market in question.

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